# structures (see also Fig. 1)

Fractional coordinates of representative atoms are also given.



(a) Half of the SiO<sub>4</sub> tetrahedra of the garnet structure are retained in  $C_{12}A_7$  as Al(2)O<sub>4</sub> tetrahedra, avoiding any pairs of SiO<sub>4</sub> related by a centre of symmetry.

(b) Those sites from which SiO<sub>4</sub> have been omitted are available for occupancy by the X atoms of  $C_{12}A_7$ , e.g. Cl in this case.

(c) Al of garnet is retained in  $C_{12}A_7$  but its coordination is reduced from octahedral to tetrahedral by replacing triplets of garnet O omitted as indicated in (a) above by a single O(2).

(d) Ca of garnet is retained in  $C_{12}A_7$  but again as a result of the omission of SiO<sub>4</sub> as in (a) above its coordination by oxygen is reduced from 8 (cubic) to the distorted sixfold coordination described by Williams (1973). Occupancy of the 'empty' SiO<sub>4</sub> site by X (here Cl) as in (b) above will then restore some of the symmetry of the Ca coordination again exactly as described by Williams (1973).

\* Data from Wyckoff (1968).

Table 3. Distribution of atoms in the garnet and  $C_{12}A_7$  Glasser, 1987). These properties combine with high thermal stability;  $Ca_{12}Al_{14}O_{33}$  melts at ~1670 K. However, the structure type has hitherto appeared to be confined to Ca, Al and O, with only limited OH, F and Cl inclusion. The present study shows that further chemical tailoring is possible, leading to the synthesis of materials with potentially useful properties.

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#### References

- AHMED, F. R., HALL, S. R., PIPPY, M. E. & HUBER, C. P. (1966). NRC Crystallographic Programs for the IBM 360 System. National Research Council of Canada, Ottawa, Canada.
- BARTL, H. (1969a). Neues Jahrb. Mineral. Monatsh. 91, 397-404.
- BARTL, H. (1969b). Neues Jahrb. Mineral. Monatsh. 91, 404-413.
- BUSSEM, W. & EITEL, A. (1936). Z. Kristallogr. 95, 175-188.
- FISCHER, R. X. (1982). STRUPLO82. A Fortran Program for the Graphical Display of Crystal Structures. Univ. of Mainz, Federal Republic of Germany.
- HENTSCHEL, G. (1964). Neues Jahrb. Mineral. Monatsh. 1, 22-29.
- HOSONO, H. & ABE, Y. (1987). Inorg. Chem. 26, 1192.
- ILYUKHIN, V. V., NEVSKY, N. N., BICKBAU, M. J. & HOWIE, R. A. (1977). Nature (London), 269, 397-398.
- International Tables for X-ray Crystallography (1974). Vol. IV. Birmingham: Kynoch Press. (Present distributor D. Reidel, Dordrecht.)
- JEEVARATNAM, J., DENT GLASSER, L. S. & GLASSER, F. P. (1962). Nature (London), 194, 764-765.
- KLEMM, W. A., JAWED, I. & HOLUB, K. H. (1979). Cem. Concr. Res. 9, 489-496.
- SINGH, V. K. & GLASSER, F. P. (1987). Ceramics International. In the press.

SMART, R. M. & ROY, D. M. (1979). Cem. Concr. Res. 9, 269-274. WILLIAMS, P. P. (1973). Acta Cryst. B29, 1550-1551.

WYCKOFF, R. W. G. (1968). Crystal Structures. Vol. 4, p. 159. New York: Interscience.

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## Crystal Structure of KLiSO<sub>4</sub> at 200 K: a Neutron Diffraction Study

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**Abstract.**  $M_r = 142.10$ , trigonal, P31c, a = 5.129, c = 8.639 Å,  $V = 196.82 \text{ Å}^3$ ,  $D_r =$ Z = 2,2.397 g cm<sup>-3</sup>,  $\lambda$ (neutron) = 1.176 Å,  $\mu_c = 0.74$  cm<sup>-1</sup>, T = 200 K. Refinement for the ordered structure with 593 reflections with  $(\sin\theta)/\lambda \le 0.57$  Å<sup>-1</sup> gave  $R(F^2) =$ 0.055. The study has revealed merohedral twinning of

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the crystal about (110), with twin components in nearly equal proportion, and large thermal vibration for the oxygens. Displacing the oxygens into three disordered positions gave a somewhat better R factor.

Introduction. The crystal structure of KLiSO<sub>4</sub> at room temperature was determined by Bradley (1925). It crystallizes in space group P63 (F1 phase). In recent years much work has been done to study the behaviour

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of the oxygens which form the tetrahedra. In the ideal structure model these oxygens are ordered, but exhibit unusually high temperature factors. Karppinen, Lundgren & Liminga (1983) explained the anomalous temperature factors as the results of highly anharmonic thermal motion of the oxygens. By contrast, Bhakay-Tamhane, Sequeira & Chidambaram (1984) treated O(1) atoms, which form the vertices of the tetrahedra, as occupying threefold split positions with occupancy of 1/3. Schulz, Zucker & Frech (1985) found that both oxygens show large and strongly anharmonic thermal motion as well as static disorder. They explained the thermal vibration as the coupled rotational vibration of the oxygens around the S atoms.

KLiSO<sub>4</sub> undergoes several phase transitions. A phase (F2) has been reported in a temperature range of about 250-180 K. Bansal, Deb, Roy & Sahni (1980), in their Raman scattering experiment, proposed space group P31c, caused by cooperative reorientation of SO. tetrahedra. Tomaszewski & Lukaszewicz (1982) assume space group  $P6_3mc$  to explain their X-ray diffraction data. Holuj & Drozdowski (1981) found in an EDR study that this phase may have an incommensurate structure. Bhakay-Tamhane, Sequeira & Chidambaram (1985) found, in a single-crystal neutron diffraction study in this temperature range, that there are two types of domain in P31c in nearly equal amounts. In their experiment, the F2 phase was always mixed with the F1 phase, and no structural parameters were presented.

The phase transition between F1 and F2 has been found to exhibit a rather large hysteresis. In the cooling cycle the transition occurred at about 205 K while in the heating cycle it appeared at about 250 K (Bansal *et al.*, 1980; Tomaszewski & Lukaszewicz, 1982, 1983; Bhakay-Tamhane *et al.*, 1985). Tomaszewski & Lukaszewicz (1983) found that in the cooling cycle the F2phase is always mixed with F1 phase until it transforms to another phase (F3), and pure F2 phase can be obtained only in the heating cycle from F3 to F2. Bhakay-Tamhane *et al.* (1985), in their single-crystal neutron diffraction study, could not get a pure phase even in the heating cycle.

This study was carried out to determine the precise structure of KLiSO<sub>4</sub> in the F2 phase.

**Experimental.** The specimen crystal  $(0.18 \times 0.21 \times 0.31 \text{ cm})$  was cut from a large crystal. Low-temperature data were collected using a refrigerator on the P32 four-circle diffractometer at SILOE reactor in Centre d'Etudes Nucleaires de Grenoble, France. A Cu monochromator was used,  $\lambda = 1.176 \text{ Å}$ . The sample was cooled to 150 K for an hour, and then heated slowly to 200 K. Lattice constants determined from 19 centred single-crystal reflections by least-squares refinement of the orientation matrix.  $\omega$  scan mode up to  $(\sin\theta)/\lambda = 0.57 \text{ Å}^{-1}$ . 660 reflections  $(\pm h, -k, \pm l)$  were

collected, of which 134 are independent. The mutual agreement factor for the intensities of a standard reflection, which was repeated after every 40 reflections, was better than 0.02. The Friedel pairs were averaged; those of 21 reflections which were observed negative were retained as negative. 593 reflections were used for the structure refinements.

Data reduction was carried out using COLL5N (Lehmann, 1977, unpublished). An absorption correction was made ( $\mu_c = 0.74$  cm<sup>-1</sup>) using DISTAE (modified by Tasset from DISTAO, 1981, unpublished). PRIME computer, installed at CENG DRF, France, was used for this work. All the programs mentioned above were modified by one of the authors (Boucherle) for PRIME.

No satellite reflections are observed in the diffraction data, so the incommensurate structure should be ruled out. The intensity of the 111 reflection, which is rather strong at room temperature, is zero at 200 K. It indicates that no F1 phase is mixed with F2 phase. The systematic absences are consistent with space groups P31c and  $P6_3mc$ .

The refinement of the ordered structure in space group  $P6_3mc$ , which was assumed by Tomaszewski & Lucaszewicz (1982), could not converge satisfactorily. In general, there are two positions related by a (110) mirror plane in one cell, each of which corresponds to space group  $P6_3$ . O(2), which is at the general position, is disordered at these two positions with equal occupancy. For this disordered model, the structure was refined with isotropic extinction and anisotropic temperature factors. It gave a rather high residual  $R(F^2) = 0.280$ .

At this stage, the space group P31c was adopted in the refinements. Using the model proposed by Bansal *et al.* (1980), the refinement with isotropic extinction and isotropic temperature factors gave  $R(F^2) = 0.237$ .

In our diffraction data, the differences between the intensities I(hkl) and I(hkl) are much smaller than that calculated in space group P31c. It hinted that the merohedral twinning of the crystal at 200 K, found first by Bhakay-Tamhane *et al.* (1985), should be considered. Two models (A and B), reflected by the (110) mirror plane in the space group P31c, can be obtained by the cooperative reorientation of SO<sub>4</sub> tetrahedra in space group P6<sub>3</sub> (Fig. 1). During the phase transition from F1 to F2, two domains, corresponding to models A and B respectively, are formed. The intensity of each reflection is the sum of the intensities of two domains:

$$F^{2}(hkl) = (1 - f)F^{2}_{A}(hkl) + fF^{2}_{B}(hkl), \qquad (1)$$

where f is the fraction of model B in the crystal.

The full-matrix least-squares program ORXFLS4 [a modified version of ORFLS (Busing, Martin & Levy, 1962)] was used in the structural refinements. In order to refine the twinned crystal, this program was modified by one of the authors (Zhang). The fraction f

was introduced into ORXFLS4 as an extra parameter. The calculation of intensity was carried out according to (1). 0.5 was chosen as the initial value of f. The positional parameters in  $P6_3$  reported by Schulz *et al.* (1985) were adopted as the starting parameters. With isotropic extinction and isotropic temperature factors the refinement of the structure reduced the residual dramatically to 0.072. Because the diffraction data show anisotropic extinction, a type II anisotropic extinction correction was made. With anisotropic temperature factors the refinement converged at  $R(F^2)$ = 0.055 with f = 0.464 (3). Altogether 28 variables were refined in the last cycle. The elements of the extinction tensor are

$$Z(11) = 3.9$$
 (5)  $Z(22) = 3.9$  (5)  $Z(33) = 20.6$  (32)  
 $Z(12) = 1.1$  (4)  $Z(13) = -1.4$  (8)  $Z(23) = 0.5$  (7).

The crystallographic parameters, r.m.s. displacements along the principal axis of thermal vibration, and selected bond lengths and angles are shown in Tables 1, 2 and 3 (ordered structure), respectively.\*

The temperature factors of oxygens are unusually high and strongly anisotropic, rather similar to those at room temperature reported by Schultz *et al.* (1985). Both O(1) and O(2) show large motion normal to the S-O bonds.

On changing the position of O(1) from the threefold axis with other parameters fixed, a minimum R factor was obtained at about x = 0.32, y = 0.67, z = 0.04. Using the split O(1) model the structure was refined with anisotropic temperature factors (34 variables were refined). The R factor was reduced a little:  $R(F^2)$ = 0.052. By splitting O(2) into three positions with occupancy of 1/3 for each one, with other parameters fixed, the R factor was reduced further. In order to

\* Lists of structure factors have been deposited with the British Library Document Supply Centre as Supplementary Publication No. SUP 44551 (23 pp.). Copies may be obtained through The Executive Secretary, International Union of Crystallography, 5 Abbey Square, Chester CH1 2HU, England.



Fig. 1. View of  $KLiSO_4$  in space group P31c in the direction [001]. The shaded circles are separated from open circles by c/2. The solid triangles and the dashed triangles represent the orientations of  $SO_4$  tetrahedra in models A and B, respectively.

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Table 1. Atomic coordinates and thermal parameters

$\boldsymbol{B}_{eq} = \frac{4}{3} \sum_{i} \sum_{j} \beta_{ij} \mathbf{a}_{i} \cdot \mathbf{a}_{j}.$							
	x	у	z	$B_{\rm iso}/B_{\rm eq}({\rm \AA}^2)$			
Ordered	structure			••••			
к	0	0	0	1.27 (5)			
Li	ł	<del>2</del>	0.8156 (6)	0.89 (10)			
S	1	2	0-2049 (3)	0.74 (7)			
O(1)	1 1	ž	0.0350 (3)	2.44 (5)			
O(2)	0.0591 (2)	0.6589 (2)	0.2595 (3)	1.65 (3)			
Disorder	ed structure						
К	0	0	0	1.27 (3)			
Li	1	2	0.8162 (5)	1.05 (5)			
S	1 1	2	0.2052 (3)	0.70 (4)			
O(1)	0·2824 (7)	0.6571 (14)	0.0369 (5)	0.82 (4)			
O(21)	0.0603 (10)	0.6811 (10)	0.2595 (11)	0.65 (12)			
O(22)	0.0715 (7)	0.6502 (11)	0.2844 (7)	1.11 (10)			
O(23)	0.0461 (8)	0.6433 (10)	0.2414 (6)	0.44 (11)			

Table 2. Selected r.m.s. displacements along theprincipal directions of vibration and the correspondingdirection cosines in a Cartesian system

Ordere	d structure			
	$U(\mathbf{A})$	cosa	cosβ	cosy
<b>O</b> (1)	$U_{11} = 0.205$	1.0000	0.0000	0.0000
	$U_{22} = 0.205$	0.0000	1.0000	0.0000
	$U_{33}^{\prime \prime} = 0.093$	0.0000	0.0000	1.0000
O(2)	$U_{11} = 0.098$	0.8755	-0.3662	-0.3153
	$U_{22} = 0.125$	0.4080	0.9098	0.0760
	$U_{11} = 0.193$	0.2590	-0.1952	0.9460

The x axis of the Cartesian system is perpendicular to the bc plane in the cell, y and z are in the directions of **b** and **c** respectively.

Table 3. Selected bond lengths (Å) and angles (°)

Ordered structure		Disordered structure		
S-O(1)	1.47	S-O(1)	1.47	
S = O(2)	1.46	S-O(21)	1.51	
		S-O(22)	1.47	
		S-O(23)	1.45	
O(1) - S - O(2)	109	$O(1) - S - O(21^{i})$	111	
		O(1)-S-O(22)	108	
		O(1)-S-O(23 <sup>ii</sup> )	108	
$O(2) - S - O(2^{i})$	110	O(21 <sup>i</sup> )-S-O(22)	102	
		O(21 <sup>i</sup> )-S-O(23 <sup>ii</sup> )	119	
		O(22)-S-O(23 <sup>ii</sup> )	109	
LiO(1 <sup>iii</sup> )	1.90	Li–O(1 <sup>iii</sup> )	1.92	
Li-O(2 <sup>iv</sup> )	1.93	Li-O(21 <sup>v</sup> )	1.97	
		Li-O(22 <sup>iv</sup> )	1.91	
		$Li - O(23^{vi})$	1.91	

Labeling of positions: (i) 1-y, 1+x-y, z; (ii) -x+y, 1-x, z; (iii) x, y, 1+z; (iv) -x, -x+y, z+0.5; (v) 1+x-y, 1-y, z+0.5; (vi) y, 1+x, z+0.5.

avoid too many parameters being refined, isotropic temperature factors were adopted in the refinements of the O(2) disordered structure. The refinement with 29 variables gave  $R(F^2) = 0.051$ , f = 0.464 (3). The positional and thermal parameters are shown in Table 1, selected bond lengths and angles in Table 3 (disordered structure).

Scattering lengths for K, Li, S and O were taken from Sears (1984). An IBM-158 computer, installed at the Institute of Physics, Chinese Academy of Sciences, Beijing, was used for the structure refinements.

**Discussion.** As mentioned before, the phase transition from  $P6_3$  to P31c is associated with the cooperative reorientation of the SO<sub>4</sub> tetrahedra (Bansal *et al.*, 1980). The rotation of one of the two tetrahedra in one cell in  $P6_3$  causes two types of domain in P31c, in equal proportions. No matter whether the crystal is merohedrally twinned or not at room temperature, the twin fraction in P31c should be the same. In this experiment the twin fraction f is 0.464 (4); the small difference of f from  $\frac{1}{2}$  may be caused by the inhomogeneous distribution of the defects in the crystal.

The splitting of both O(1) and O(2) into three sites means that the rigid  $SO_4$  tetrahedra are disordered in three positions with equal probability, which is rather similar to the results at room temperature reported by Schulz *et al.* (1985).

With the method reported by Tomaszewski & Lukaszewicz (1983), pure P31c phase was obtained in our experiment. Using a similar method, Bhakay-Tamhane *et al.* (1985) did not obtain a pure phase in a single crystal grown by themselves. During the transformation from F1 to F2, one of the two tetrahedra turns about 57°. This rotation may be sensitive to the concentration of defects in the crystal, which depends on the growth conditions.

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#### References

- BANSAL, M. L., DEB, S. K., ROY, A. P. & SAHNI, V. C. (1980). Solid State Commun. 36, 1047–1050.
- BHAKAY-TAMHANE, S., SEQUEIRA, A. & CHIDAMBARAM, R. (1984). Acta Cryst. C40, 1648–1651.
- BHAKAY-TAMHANE, S., SEQUEIRA, A. & CHIDAMBARAM, R. (1985). Solid State Commun. 53, 197–200.
- BRADLEY, A. J. (1925). Philos. Mag. 49, 1225-1237.
- BUSING, W. R., MARTIN, K. O. & LEVY, H. A. (1962). ORFFE. Report ORNL-TM-306. Oak Ridge National Laboratory, Tennessee, USA.
- Holuj, F. & Drozdowski, M. (1981). *Ferroelectrics*, **36**, 379–382. Karppinen, M., Lundgren, J. O. & Liminga, R. (1983). *Acta*
- *Cryst.* C**39**, 34–38.
- SCHULZ, H., ZUCKER, U. & FRECH, R. (1985). Acta Cryst. B41, 21-26.
- SEARS, V. F. (1984). Thermal-Neutron Scattering Lengths and Cross Sections for Condensed-Matter Research. Chalk River Nuclear Laboratories, Ontario, Canada.
- TOMASZEWSKI, P. E. & LUKASZEWICZ, K. (1982). Acta Cryst. A38, 729-733.
- TOMASZEWSKI, P. E. & LUKASZEWICZ, K. (1983). Phase Transitions, 4, 37-46.

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### Structure of Yttrium Nitrate Trihydrate

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Abstract.  $Y(NO_3)_3.3H_2O$ ,  $M_r = 328.97$ , triclinic,  $P\overline{I}$ , a = 6.946 (2), b = 7.323 (1), c = 10.948 (1) Å, a =71.47 (1),  $\beta = 78.38$  (1),  $\gamma = 67.64$  (1)°, V =486.2 (2) Å<sup>3</sup>, Z = 2,  $D_x = 2.246$  g cm<sup>-3</sup>,  $\lambda$ (Cu Ka) = 1.54184 Å,  $\mu = 93.7$  cm<sup>-1</sup>, F(000) = 324, T =296 (1) K, R = 0.028 for 1878 unique observed reflections. The compound was prepared by thermal decomposition of crystals of Y(NO\_3)\_3.6H\_2O (Merck, Art. 12516) at 382 K. Three bidentate nitrate groups and three water molecules produce a nine-coordinated Y atom. The coordination polyhedra are linked by hydrogen bonds into a three-dimensional network. **Introduction.** Differential thermal analysis of  $Y(NO_3)_{3,-}$ 6H<sub>2</sub>O shows two strong endothermic peaks at 361 and 378 K. At the first peak a complex of  $[Y(H_2O)_5-(NO_3)_2][Y(H_2O)_2(NO_3)_4]$  (Radivojević, Milinski, Ribár & Lazar, 1984) arises and at the second one the title compound is formed which has been grown from the melt of hexahydrate at 382 K. The crystals were sealed into Lindemann capillary tubes.

**Experimental.** A crystal *ca*  $0.20 \times 0.25 \times 0.40$  mm was mounted on a CAD-4 diffractometer equipped with graphite monochromator. Scan range  $3.0 < 2\theta < 150^\circ$ ,

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